Vector Analysis

$$\vec{A} = \hat{a}|\vec{A}| = \hat{a}A$$ notation: arrow (or bold) = vector
hat = unit vector

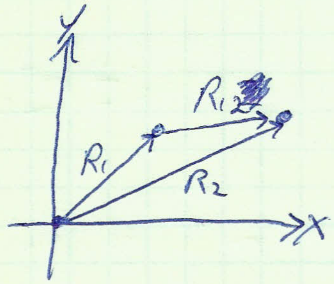
- In cartesian coordinates,

$$\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$$

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

- position vector from origin to point 1

$$\vec{R_1} = \overrightarrow{OP_1} = \hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1$$



- Distance vector from point 1 to point 2

$$\vec{R_{12}} = \overrightarrow{P_1P_2} = \hat{x}(x_2 - x_1) + \hat{y}(y_2 - y_1) + \hat{z}(z_2 - z_1)$$
$$= \vec{R_2} - \vec{R_1}$$

- Product of vector and scalar

$$\vec{B} = k\vec{A} = \hat{x}\underbrace{(kA_x)}_{B_x} + \hat{y}\underbrace{(kA_y)}_{B_y} + \hat{z}\underbrace{(kA_z)}_{B_z}$$

- Dot product of two vectors

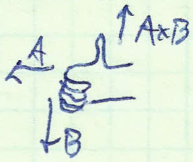
$$\vec{A}\cdot\vec{B} = AB\cos\theta_{AB}$$ → result is a scalar

$$\vec{A}\cdot\vec{A} = |\vec{A}|^2 = A^2$$

- Cross product of two vectors

$$\vec{A}\times\vec{B} = \hat{n}AB\sin\theta_{AB}$$ → result is a vector

follows right hand rule



$$A\times A = 0$$

determinant

$$A\times B = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(A_yB_z - A_zB_y) + \hat{y}(A_zB_x - A_xB_z) + \hat{z}(A_xB_y - A_yB_x)$$

Orthogonal Coordinate Systems

- Cartesian coordinates

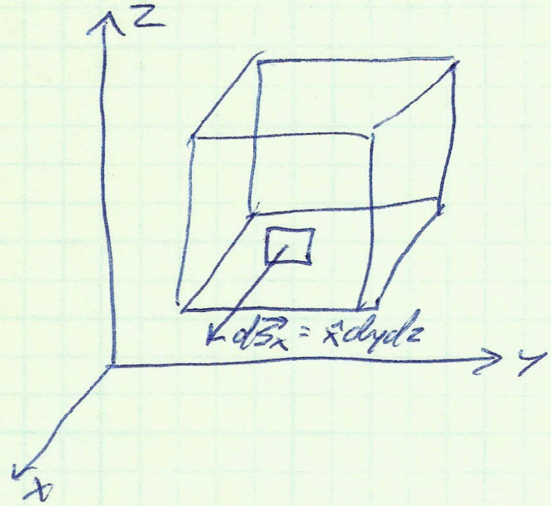
$\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$

note $\hat{x}\times\hat{y} = \hat{z}$,

$\hat{y}\times\hat{z} = \hat{x}$,

$\hat{z}\times\hat{x} = \hat{y}$

Same rotation works for cylendrical and spherical too



$d\vec{l} = \hat{x}dx + \hat{y}dy + \hat{z}dz$

$d\vec{s}_x = \hat{x}dydz$, $d\vec{s}_y = \hat{y}dxdz$, $d\vec{s}_z = \hat{z}dxdy$

$dV = dxdydz$

- Cylendrical coordinates

$\vec{A} = \hat{r}A_r + \hat{\phi}A_\phi + \hat{z}A_z$

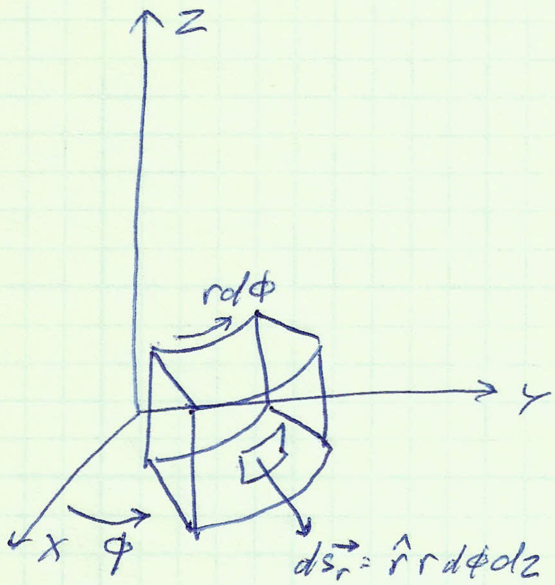
$d\vec{l} = \hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz$

$d\vec{s}_r = \hat{r}rd\phi dz$

$d\vec{s}_\phi = \phi drdz$

$d\vec{s}_z = \hat{z}rdrd\phi$

$dV = rdrd\phi dz$



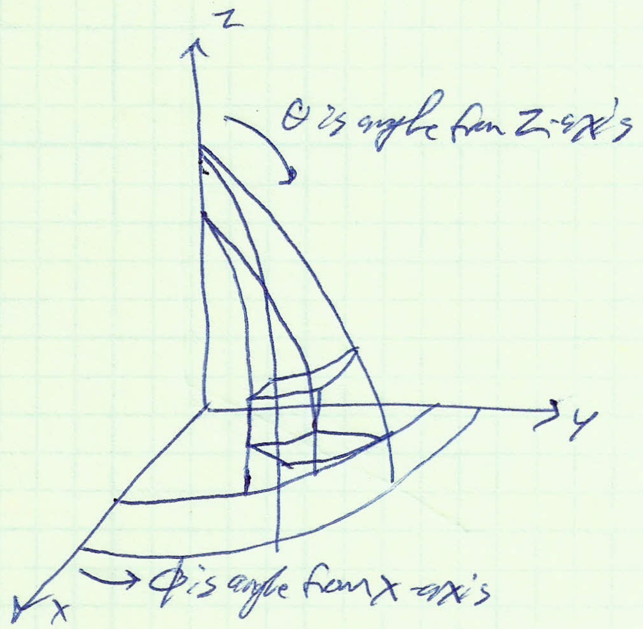
ϕ is angle from x-axis

note that length along ϕ is $r\Delta\phi$

One complete circle is $2\pi r$, as expected

- Spherical Coordinates

$$\vec{A} = \hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$$



$$d\vec{l} = \hat{R}\,dR + \hat{\theta}R\,d\theta + \hat{\phi}R\sin\theta\,d\phi$$

$$d\vec{S}_R = \hat{R}R^2\sin\theta\,d\theta\,d\phi$$

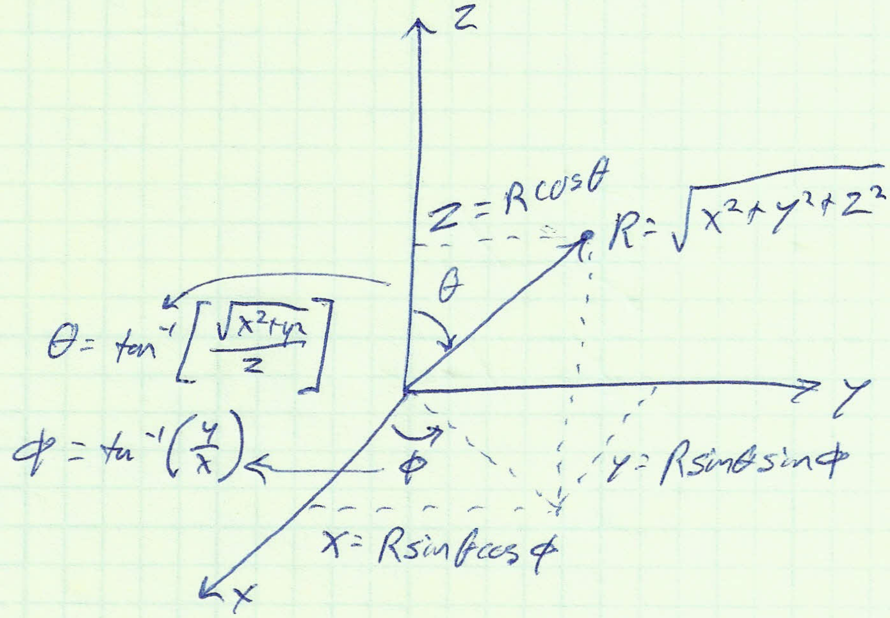
$$d\vec{S}_\theta = \hat{\theta}R\sin\theta\,dR\,d\phi$$

$$d\vec{S}_\phi = \hat{\phi}R\,dR\,d\theta$$

$$dV = R^2\sin\theta\,dR\,d\theta\,d\phi$$

- θ works similarly to ϕ in cylindrical
  - distance along θ is $R\Delta\theta$
- lengths and areas and volumes are compressed by $\sin\theta$
  - note that lengths, etc go to zero at $\theta = 0$

- Transformations – just simple geometry



Other transformations can be obtained in a similar way